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The Trade-Offs of Fighting and Investing: A Model of the Evolution of War and Peace[1]

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International competition occurs in many different forms. Just as a state would be in danger if it allowed its opponent to gain a military advantage, one that falls behind a rival in an economic contest similarly faces risks. States must weigh the trade-offs between economic and military growth, as well as deciding on the best strategy to follow should war erupt. We use a formal, dynamic model to explicitly capture the trade-offs that states face in their search for security and dominance. The deductions from the model demonstrate that by considering the long-run results of a peacetime rivalry, weaker states might conclude that their only hope of winning or surviving a rivalry lies in fighting a counterforce war, explain why and how stalemates evolve during counterforce wars, and indicate that targeting industrial objectives shortens the duration of wars.

Keywords competition, dynamic model, strategy, trade-offs, war

States compete with their rivals in a variety of ways. They may endure long-term contests for economic superiority, use aggressive military spending to drain enemy budgets, bomb factories and infrastructures, or wage bloody battles. How do states weigh these options and ultimately choose a strategy for winning a rivalry? We seek to answer this question by examining the variety of trade-offs and considerations that opposing states face. For example, each state must balance military development with economic growth, while also reacting to how its opponent makes spending decisions. Although economic investment yields long-term resource growth, it might do so at the risk of losing a more immediately pressing military competition. During wartime, states must address additional, but similarly vexing, allocation concerns. Concentrating destructive efforts on the enemy's military forces might ultimately produce an advantage in the event of head-to-head combat, but if the opponent does likewise, the two sides' economies might be able to sustain a long, cumulatively costly

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war. And if rivals instead target one another's industrial facilities, their resource bases will suffer, but the resulting shorter period of direct engagement is a desirable side-effect.

To explain these varied tradeoffs—short term versus long term gains, military spending versus economic investment, and counterforce versus industrial warfare—we offer a set of three formal models. Each model represents the dynamic interplay among two opposing states' aggregate resources and military expenditures given particular macro strategies for pursuing interstate competition. In the baseline model, the two states experience a peaceful rivalry. In the second model, they undertake a traditional war, with each side's military used to destroy the other. And in the third, the opponents fight an industrial war, targeting each others' underlying economic resources. These models allow us to better understand the relationship between the methods for managing rivalries and wars and the circumstances preceding, during, and resulting from wars and peacetime contestations. As a result, we are able to answer questions about why weak states fight, why military stalemates occur, and how wars can be abbreviated. Because answers to these questions reveal something about states' preferences for various outcomes from peacetime and wartime strategies, they also speak to the question of why states go to war in the first place. The decision to enter war, for example, is based on the projected long-term outcomes from following various strategies, which include maintaining a peaceful rivalry.

More broadly, our essay represents an effort to gain a fuller understanding of how and why strategies for managing peace and war matter. In doing so, we take cues from existing black box scholars. For instance, we adopt a dynamic approach, as recommended by Maoz and Siverson (2008, this issue) in their review of the management of war literature, and consider the over-time processes and outcomes associated with those strategy options.[2] Additionally, we note that particular types of strategies have domestic consequences, perhaps as a result of the public's willingness to sustain additional casualties (Sullivan, 2008, this issue; Gartner, 2008a, this issue) and therefore assume that military competition is subject to constraints from within the state. Thus, we seek to address questions familiar to students of the black box of war while also connecting these questions back to the broader conflict literature on the many tradeoffs inherent in states' competitive strategies.

After a brief review of the trade-offs literature and the presentation of rationale for our modeling approach, the paper presents three models, with the specifications being generated by linking general theoretical arguments to mathematical representations. Because its structure serves as the building blocks for the counterforce and industrial war models, we first model the peacetime rivalry circumstance. The counterforce and industrial warfare models follow. Each of the three models represents an ideal type of contestation—one in which states use no military force, one in which states only use military force to counter the opponent's military capabilities, and one in which states use military force to undermine only the opponent's resource and production base. After the three models are constructed, we turn to their analysis. We investigate their equilibria and run a series of numerical simulations to project over-time patterns in economic and resource allocations under each of the three scenarios. Based on these analyses of the three archetype models, we offer hypotheses concerning the conditions under which states should prefer the various types of contests, connections between the type of war and its duration, and the relationship between allocation portfolios and war outcomes.

[2]Our mathematical model is a necessary first step toward the more complicated formalization of a process Maoz and Siverson (2008, this issue) see as the ideal representation of strategy dynamics. In particular, they are interested in a situation in which mid-war shifts in strategy can occur.

Background

Trade-offs inherent in the choices states make concerning peacetime competition, the use of force, and tactical strategies have been examined by military historians, defense economists, political economists, and international conflict scholars. Particular trade-offs of interest include arms versus consumption (Powell, 2006), appropriation of enemy resources by fighting versus peaceful production of goods at home (Hirshleifer, 1988), choosing battlefield targets versus economic targets (Bellany, 1999), and the power to hurt the enemy versus the ability to withstand costs imposed by the enemy (Slantchev, 2003). Using the overarching theme of fighting versus investing, we examine these trade-offs together, considering both those that underscore the war or peace dimension as well as those inherent in selecting a tactical strategy of fighting. Powell points to the need for more integrative assessments of such value exchanges:

> Interestingly, models of conflict developed by economists generally do include a resource trade-off but not an explicit decision to fight or attack . . . whereas those developed by political scientists typically do include an explicit decision to attack but not a resource trade-off. (2006, 193, fn 58)

Dynamic Models

In order to represent the trade-offs of different types of interstate competition, we construct a dynamic model of competition, with variants for different types of wars that states can fight. Dynamic models are powerful tools for understanding social phenomena. They have been used to understand a variety of over-time processes such as arms races (Richardson, 1960), the development of interpersonal relationships (Felmlee & Greenberg, 1999), the diffusion of war (Kadera, 1998), the emergence of systemic patterns of international friendships and enmities (Lee et al., 1994), and the evolution of interstate rivalries (Morey, 2006).

Dynamic models are especially well suited to the class of phenomena we seek to investigate. In particular, they can express a variety of relationships between stimuli and behavior. The researcher establishes the precise nature of the relationships using current theory and empirical findings as guides. That is, dynamic models are fairly flexible and can be used to capture a variety of theoretical perspectives. One natural extension of this is that dynamic models can nicely combine different pieces of information together to form a more complete story. Consider, for example, scenarios in which two variables impact each other. Suppose we know that war affects domestic politics (Arena, 2008; this issue, Gartner, 2008a, this issue; Sullivan, 2008, in this issue) and domestic politics alters war behavior (Lai & Morey, 2006; Lai & Slater, 2006). In the fighting and investing scenario, we know that there are several bi-directional relationships between various pairs of variables. Notably, each state's military spending is spurred by the other's military spending, and overall resources constrain a state's military spending and military spending dampens growth in total resources. While it is typically difficult for a single study to fully explore these feedback relationships, dynamic models facilely handle such complexities (Brown, 2007). Moreover, dynamic models' versatility allows them to integrate multiple explanations of behavior to show how each explanation is actually a special case of a more general theory. This is the case in Kadera's (2001) work, which combines the logic of the seemingly opposed Balance of Power and Power Transition theories to show how each is correct under certain conditions. In this paper, we integrate multiple trade-offs into a single model.

One possible criticism is that dynamic models do not capture the strategic interactions between states. While this is true to a certain degree, we prefer to take the view that "[e]ach approach to modeling has its own advantages and disadvantages" (Brown, 2007, 3). Dynamic models are able to elaborate the environment within which leaders interact and how that environment changes based on the actions the actors take. Instead of assuming the strategic setting remains unchanged by actor behavior, dynamic models allow us to explore how the relationship between states will change based on the decisions made at any point in time. In this way, dynamic models are able to help define the initial conditions that game theoretic models assume and how such combinations might arrive. They also allow for the comparison of future outcomes based on different decisions a leader could make in the present. Thus, instead of maximizing short-term utility, dynamic models allow us to analyze the long-term impact of behavior and compare across different potential realities to determine the best current policy. In this way, dynamic models are not competitors with game-theoretic models; they are best seen as complementary with a focus on long-term implications of behavior.

An energetic and optimistic critic might direct us to develop a model that is both dynamic and game-theoretic. The mathematics of such a model are likely to be too unmanageable for our first step toward integrating multiple trade-offs in a time-dependent mathematical structure. Similarly, Maoz and Siverson suggest that starting from a game-theoretic model and incorporating dynamics may be intractable:

> [M]odels of war bargaining . . . are complex to begin with. Suggesting a dynamic modeling of the interaction of strategy and bargaining may render them unwieldy. (Maoz and Siverson, 2008, this issue), p. 171)

In sum, we find the dynamic approach to be best suited to the problem at hand. We now turn to the development of the model itself.

Modeling Peacetime Rivalry

States can gain security through gathering enough resources so that a rival does not attempt to interfere with its actions (Mearsheimer, 2001, ch. 2). If enough resources are acquired, the state will be just as free to implement its favored policies as it would be if the rival were destroyed in war. This resource-based competition is ever-present between rivals and forms the heart of the security dilemma.

Our baseline model starts with two assumptions about resource growth. First, both economic and military growth require investment. All things equal, the more invested, the faster they grow. Second, there is a guns versus growth trade-off built into the decision to allocate resources for military production. The more you invest in the military, the less you can invest in the economy (Knorr, 1970, 32). Military expenditures come with the opportunity cost of reduced economic spending. Powell reminds us that fighting is costly, but so is "procuring the means needed to fight" (2006, 195).

Working from these assumptions, we start to specify nation *i*'s resource growth as a function of the amount of resources *i* invests in economic growth versus the amount it invests in military preparations for fighting. Let:

$$r_i' = \alpha_i(r_i - m_i) \tag{1}$$

where:

r_i is nation *i*'s aggregate resource base, $0 \leq r_i$
m_i is nation *i*'s level of military spending, $0 \leq m_i \leq r_i$, and
α_i is a positive parameter.

In other words, all of nation i's resources, except those it spends on its military, generate future resources. The amount invested by i is multiplied by i's economic growth multiplier (α). The economic growth multiplier represents how well i is able to channel nonmilitary spending into actual economic growth. We can also think of α-as the bang i gets for every buck it spends on growth or as the amount of growth resulting from the investment of one unit of resources. A more efficient economy will have a higher value for this parameter. Thus, a state with a large economic growth multiplier can maintain greater levels of growth while spending more on other areas, namely the military. This, of course, is a simplified model of growth, as it assumes states only invest in growth or the military. It also ignores possible spin-off effects from military research, which might increase economic growth in the long term. In this case, we feel that modeling the arms versus growth decision captures the heart of the dilemma facing states without overly complicated the model. Certainly domestic politics or leader preferences may interfere and redirect spending towards other activities. When this is the case, the model will predict what would be best outcomes had resources not been diverted. However, when faced with the type of competition we seek to model, it is most likely that states will focus on long-term survival, which will move a state's decision calculus back toward the simple dichotomous choice we have constructed.[3]

Because national growth rates are not unlimited, we also set an upper bound to nation i's resource growth. This limit (K_i) represents i's maximum level of economic development and can be thought of as its carrying capacity (Edwards & Penney, 1985; Mesterton-Gibbons, 1989; Cohen, 1995). Once a state has reached its carrying capacity, future investment into resources will not yield any more growth. The carrying capacity, a concept borrowed from ecology, is the upper limit of units that can be supported by a given environment. Each state can only support a certain level of economic development given its current holdings.[4] In our particular case involving a state allocating resources, the upper limit on resource growth is reached at K_i–m_i, the total capacity less what has been set aside for the military. To reflect this limit on growth, we modify Equation 1 as follows:

$$r_i' = \alpha_i(r_i - m_i)\left(1 - \frac{r_i}{K_i - m_i}\right) \tag{2}$$

According to Equation 2, a state with low levels of development will achieve higher rates of growth from its investment than a more developed state with the same amount of investment. This construction is consistent with theories of growth found in international relations research, most of which predict an S-shaped patter of state development. States with very low development levels are unable to grow quickly because of a lack of resources. However, sates with a high level of development also suffer slower growth as they approach the upper limits of their development potential. With high yield projects completed already, state growth slows as more expensive and lower yielding projects are the only options left for development.

The construction of Equation 2 also captures the logic of Organski and Kugler's (1977) finding of a phoenix phenomena for defeated states after a war. A defeated major power would suffer great loss, pushing it away from its development maximum; however, since it

[3]Morton (1999) also reminds us that the test of a model's assumption is not how accurately they reflect reality. Instead, the measure of an assumption is how well it helps to understand behavior.

[4]In fact, efforts to increase a state's carrying capacity drive many explanations of state expansion. For instance, Jervis (1978) argues that states seeking security may attempt to expand as a way to gain more resources that will allow them to better protect themselves. It is estimated that during the Second World War, occupied territories increased the proportion of production available to the German government by 59% (United States Strategic Bombing Survey, 1945, 21)

is a major power, the state would still have significant resources to invest in growth, allowing for a quick rise. Equation 2 moves beyond this one case of the phoenix rising to explain why low cost wars and high cost wars between smaller states will not produce a rapid post war economic explosion. Thus, Equation 2 captures many fundamental patters of post war growth, but does so in a parsimonious manner.

The final part of a state's growth equation is the cost associated with military expenditures. Not only do military expenditures reduce the resources that can be invested into resource growth, they also act as a drag on growth. This drag is a function of the amount of spending on the military as a proportion of overall resources (m_i/r_i). Troops, equipment, and weapons systems require maintenance, which produces the extra drag on resources. The understanding of the military's extra burden on the economy anchored much of President Reagan's strategy towards the U.S.S.R. The belief that the Soviet economy would be hurt at a faster rate than America's is the center of Cold War outspending theory (Commission on Integrated Long-Term Strategy, 1988). It is also at the heart of theories of nations overreaching their potential (Kennedy, 1991; Morgenthau, 1978). Nations that develop too many commitments and require too much military production eventually drive their economies to ruin. Bringing all of the parts together gives us the complete resource growth equation for a nation:

$$r_i' = \alpha_i(r_i - m_i)\left(1 - \frac{r_i}{K_i - m_i}\right) - \beta_i\frac{m_i}{r_i} \tag{3}$$

where:

β_i is a positive parameter.

There is more than one reason for a state to limit military spending, "... states sometimes limit defense spending either because spending more would bring no strategic advantage or because spending more would weaken the economy and undermine the state's power in the long run" (Mearsheimer, 2001, 37). Equation 3 captures this basic trade-off states face in their drive for security. In the 1930s, Germany confronted this issue in regards to its efforts to dominate Europe. The Third Reich could have undertaken a large and time-consuming effort to expand its resource base (r), or what is often called armament in depth, "laying the foundations of a war economy by expanding their basic industries and building up equipment for the mass productions of munitions" (United States Strategic Bombing Survey, 1945, 7). However, inasmuch as Hitler did not expect to face a united Allied front and because he focused on gaining his objectives quickly, Germany did not choose to maximize its military productive capabilities prior to war. Once Germany realized it was involved in a large-scale world war, it was forced to follow a policy of armament in width, or maximization of military production (m), within a resource base that was fixed in the short-term. This focus on armament in width over armament in depth certainly had important implications for Germany's war fighting abilities.

The rate of investment into resource growth is conditioned upon the amount of investment in the military, which means a decision rule for investment into the military is required. An action-reaction component determines shifts in nation i's military spending (Richardson, 1960). In other words, nation i's military spending is conditioned upon the level of military spending by the other state, nation j. The rationale behind such a dynamic is well known. If states allow a competitor to develop more militarily, it will be less secure as the competitor can now use its advantaged position to blackmail the state or outright conquer it. This assumption lies at the core of realist thinking and the security dilemma (Waltz, 1979; Morgenthau, 1985; Jervis, 1978). Equation (4) represents this competitive dynamic by expressing the change in nation i's military spending as a positive function of nation *j*'s military spending:

$$m_i' = \delta_i m_j \tag{4}$$

where:

δ_i is a positive parameter.

As was the case for economic growth, military spending cannot continue without bound (Bellany, 1999, 730). Military spending has its own carrying capacity, which equals the total resource base of the nation.[5] In other words, a nation cannot spend more on the military than the resources it possesses. Further, military spending slows as it approaches this upper limit. We know, for example, that as the costs of war rise, the public becomes less willing to endure additional costs (Gartner, 2008a, this issue). This supposition also demonstrates the importance of economic growth even to security minded states living under anarchy. Maximizing military power in the short run can harm military strength in the long run by limiting economic growth. Economic resources represent future military power potential. Prosperity comprises the core of what Mearsheimer (2001) terms "latent power." States must balance the exigency of military spending today against the prudent accumulation of resources that can be spent tomorrow. We modify Equation 4 to take this into account:

$$m_i' = \delta_i m_j \left(1 - \frac{m_i}{r_i}\right) \tag{5}$$

Adding the second component to Equation 4 has the effect of slowing the rate of military growth as military spending becomes a larger percent of the state's total resource base (r). All else equal, as m_i increases, the ratio of military spending to total resources, $\frac{m_i}{r_i}$, will also increase. Therefore, the proportion of resources not spent on the military, $1 - \frac{m_i}{r_i}$, decreases. As non-military spending approaches zero, growth in military spending, m_i', slows to zero. In the end, Equation 4 models the state's response to increased military spending by the rival, while Equation 5 restricts the magnitude and speed of this response give the state's resource limitations.

Collecting Equations 3 and 5 and the mirror equations for state *j* provides the system of differential equations that model state interaction during peacetime rivalry:

$$r_i' = \alpha_i(r_i - m_i)\left(1 - \frac{r_i}{K_i - m_i}\right) - \beta_i \frac{m_i}{r_i} \tag{3}$$

$$m_i' = \delta_i m_j \left(1 - \frac{m_i}{r_i}\right) \tag{5}$$

$$r_j' = \alpha_j(r_j - m_j)\left(1 - \frac{r_j}{K_j - m_j}\right) - \beta_j \frac{m_j}{r_j} \tag{6}$$

$$m_j' = \delta_j m_i \left(1 - \frac{m_j}{r_j}\right) \tag{7}$$

Equations (3), (5), (6), and (7) represent the behavior of states that are at peace but nonetheless in competition. However, the model would be of little help unless it can explain interactions under a state of war as well. To achieve this, we now offer two revised versions

[5]For now, we set aside the possibility that allies can increase a state's military capacity.

of the model. These modifications capture two tactical strategies by which states use their militaries during war to demolish one another's resources.

Modeling Counterforce Wars

Why do states shift from resource gathering to war fighting? War is the most direct form of interstate competition. Employing its military, a state can destroy a rival's ability to threaten it. With the rival neutralized, the state is free to pursue policies in its best interest.

In a traditional, or counterforce, war, a state targets the opponent's military but does not harm its industrial base (either because it cannot reach the means of production or because it finds the military targets more threatening). State *i*'s underlying allocation decision remains the same, but now its military growth is hampered by state *j*'s armed attacks. Therefore, we retain Equations 3 and 6 and modify Equations 5 and 7 as follows:

$$m_i' = \delta_i m_j \left(1 - \frac{m_i}{r_i}\right) - \rho_j m_i m_j \tag{8}$$

$$m_j' = \delta_j m_i \left(1 - \frac{m_j}{r_j}\right) - \rho_i m_i m_j \tag{9}$$

where

ρ_i and ρ_j are positive constants.

Equations 3, 6, 8, and 9 comprise a system of differential equations that capture shifts in economic and military resources of two states while they are engaged in a counterforce war.

Three features of the war component of Equations 8 and 9 warrant discussion. First, the level of damage (the amount of military put out of action) imposed by state *j* is dependent on the level of *j*'s military force (m_j). All else being equal, the larger *j*'s military, the more of *i*'s forces it should be able to knock out. Second, *j*'s military destruction is also a function of its military's efficiency (ρ_j), or its "power to hurt" (Slantchev, 2003). Just as with economic growth, militaries can be more or less efficient at ruining opposing units. During the Vietnam War, for example, the U.S.'s ability to impose costs on North Vietnam's military "was severely limited by the guerilla tactics of the Viet Cong" (Slantchev, 2003, 131). The ρ_j parameter represents the amount of military forces one unit of *j*'s military can wipe out in a given unit of time (Bellany, 1999, 730). At the same time, *i* imposes costs on *j*, and does so with efficiency ρ_i. From j's perspective, ρ_i is *j*'s ability to "bear costs in return" (Slantchev, 2003). Finally, the level of military destruction is conditioned on the interaction opportunities that exist between the two armies. In his famous work, *On War*, Clausewitz explains that the level of destruction from direct combat is not determined by the balance of power between the two armies but by the aggregate level of troop engagement:

> "A thousand men fire twice as many rounds as five hundred, but of the thousand, more will be hit than of the five hundred, for it must be assumed that the thousand will be deployed more closely" (1976, 205).[6]

[6]While Clausewitz reasons that *total losses* for both sides is a function of the *sum* of forces, we argue that the damage to *one side* is a function of the *product* of forces. Nonetheless, the underlying principle, that the magnitude of damages is produced by the interaction of two sides, is the same.

Even a relatively small army can generate a high level of destruction if it has a great number of targets. At the same time, a larger force's effectiveness can be limited by a reduced number of targets.

Clausewitz's illustration parallels ecological models, in which shifts in a prey population are determined by an interaction between the number of prey and the number of predators. It is not only the number of foxes that matter, nor how many more foxes there are than rabbits, but the frequency with which a fox can find a rabbit that is important (Mesterton-Gibbons, 1986, 602). Following these arguments, we assume that the number of military units lost is a function of the number of times the opposing forces engage. The rate of engagement is in turn assumed to be proportional to the product of the two nations' military sizes ($m_i m_j$).

Modeling Industrial Wars

An industrial war operates much the same way as a counterforce war with the exception that a state targets an opponent's resources instead of its military forces. In a resource war, the goal is to drive the enemy's resources down so that it cannot maintain a high level of military production. In this way, you win the war by eliminating the enemy's ability to make war. While there are numerous historical examples of such tactics being employed, the most well known is the Allied bombing of Germany during the Second World War (Overy, 1995, ch. 4). Allied bombers destroyed Germany's industrial base, reducing its ability to wage war and successfully invade Europe. MacArthur failed to persuade Truman to use a similar strategy in Korea, arguing that the U.S. should push north and destroy China's capacity to resupply the North Korean forces' materiel (U.S. Senate, Foreign Relations and Armed Services Committees Hearings, 1951).

Changes in nation *i* and nation *j*'s resources and military spending during an industrial war are represented by Equations 5 and 7 coupled with these adapted version of Equations 3 and 6:

$$r_i' = \alpha_i(r_i - m_i)\left(1 - \frac{r_i}{K_i - m_i}\right) - \beta_i\frac{m_i}{r_i} - \rho_j r_i m_j \tag{10}$$

$$r_j' = \alpha_j(r_j - m_j)\left(1 - \frac{r_j}{K_j - m_j}\right) - \beta_j\frac{m_j}{r_j} - \rho_i r_j m_i \tag{11}$$

As Equation 10 demonstrates, state i's resources are destroyed at a rate proportional to the military efficiency of state j (ρ_j) and the size of state *j*'s military (m_j). Further, the number of opportunities for interaction between i's resources and *j*'s military conditions the rate of destruction. The logic is that the more resources one state has, the easier it is for the enemy to damage some portion of them. This can happen due to an increased number of target opportunities or from a state's inability to defend all of its resources simultaneously.[7] Such concerns were salient during the U.S.'s recent air war over Afghanistan. Many critics of the war argued that it would serve little purpose, as there were no targets to be destroyed in Afghanistan. Whether thinking of military or economic destruction, targets must exist before they can be razed.

Studies on the effectiveness of allied bombing during World War Two support the logic of Equations 10 and 11. Initially Anglo-American bombing raids had only a negligible

[7]In order to capture the effect of targeting another state's resource base, we added $-\rho_j r_i m_i$ to Equation 6, forming Equation 10. As ρ_j, r_i, or$m_i$increases, holding the other two constant, the rate of destruction increases.

effect on German production. However, around 1943, the effect became much larger as the Royal Air Force increased resources devoted to bombing raids and as the American Air Force reached full strength in Europe (United States Strategic Bombing Survey 1945, 11). Early on, very little force was employed in bombing German targets, which resulted in meager destruction, despite the large number of potential targets. A subsequent increase in the level of forces dedicated to bombing missions augmented the effect on German production capabilities. Following the successful raids, Germany dispersed its remaining means of production, greatly reducing the bombers' ability to strike the remaining factories (United States Strategic Bombing Survey, 1945). The reduction in the number of factories (decreasing Germany's r) and the dispersal plan (decreasing the value of the Allies' ρ) reduced the amount of destruction from subsequent bombing missions.

Predictions

With the equations in place, we turn to deriving predictions regarding how rivals' economic and military resources evolve under each of the three scenarios. In a sense, comparing results across the three possible types of interstate competitions allows us to examine the processes and outcomes of different historical possibilities. This procedure serves as a type of counterfactual test. We can observe the conditions under which a state would be better off using a specific mode of contestation.

When deriving hypotheses from formal models, analytic solutions techniques are first attempted. However, the complex nature of the equations in this work limits such an approach. Known analytic tools are insufficient to find the equilibria for any of the three systems of equations. This does not mean that the models have no solutions; in fact, we are able to see equilibria result when we perform simulations. Rather, it means that appropriate mathematical solution procedures have not yet been devised.

In order to derive predictions from the model, we must instead use numerical simulations. For the simulations, random values were generated for the two states' initial military and resource levels.[8] These values were produced using the random number generation program in *Mathematica*. Each set of random values represents a hypothetical mixture of military and economic resources that could be observed in a dyad.[9] These random values were then set as the initial conditions for computer simulations (also run in *Mathematica*).[10] The numerical routine uses the system of equations to generate all subsequent and previous values of m_i, m_j, r_i, and r_j. One of the mathematical properties of differential equations is that once an initial starting point is set, all previous and future values of each variable can be uniquely determined. We plot the resulting values over time, giving us trajectories of each state's resources and military spending. Although the equations prevent the variables from taking on the value of zero when the simulations run forward, this can happen when time is reversed. Whenever the value for m_i, m_j, r_i, or r_j becomes zero in the reverse-time scenario, we stop plotting the trajectories. Each set of initial conditions was used to seed a

[8]See Appendix 1 for the parameter values of the simulations. The values for the parameters are fixed across all simulations in order to determine how changes in the variables alter dyadic behavior. If the parameters and initial conditions changed in every simulation, it would difficult to determine what was actually driving results.

[9]Aside from requiring them to be positive, we set no limits on the values of the variables. Values less than or equal to zero have no meaning in this model because it is not possible to have negative spending or resources. Once the simulations begin, we do not bound the values the variables may take on, as long as they remain greater than zero, thus we do not place any bounds on the relationship between the dyad members.

[10]Initial conditions are the values of the variables at $t = 0$, before and after $t = 0$ the values of the variables can assume any positive value.

simulation for each of the three possible sets of equations (peace, traditional war, and industrial war). The differences in outcomes observed from the equations allow for predictions to be made regarding state behavior, as well as predictions regarding the type and severity of wars.[11] The simulations we discuss below represent typical patterns that emerged from a total of about 30 sets of initial conditions (or 90 total simulations). Due to space constraints, we present graphs from only these three prototypical simulations. They represent general patterns that we observed across the entire set.[12]

Interpreting Simulation Results

Running simulations for a pair of states under peace and under the two different types of wars (with the same initial conditions) provides predictions about which strategies states will prefer. In order to prefer war to peaceful rivalry, a state would have to believe that it would be better off after a war than it would be if it maintained the peace. If the state is no better, or worse, off from fighting, we assume that it would prefer peace. For war to be a state's preferred strategy, war must leave it in a better position than maintaining normal relations would have. By "better position," we mean that as the simulations progress, the fighting state develops an advantage (or a bigger advantage) over its rival in resources, military spending, or both, whereas the peaceful state does not develop such an advantage (or develops a smaller one).

We also interpret any scenario in which a state's resources become zero as being synonymous with extinction, which is the least desirable outcome. It is worse than losing any type of contest, regardless of whether the contest is measured in aggregate resources or military spending.

When to Fight

Our first deduction from the simulations is that a state only prefers fighting to peace when it begins with a disadvantage in both resources and military.

Looking at the peaceful rivalry graph (the first column) in Figure 1, we see that state *j* is initially militarily weaker and has fewer resources than state *i*. While the initial difference between the two states appears to be small ($r_i \approx 5, r_j \approx 2.5$; $m_i \approx 1, m_j \approx .5$), state *i* does enjoy an advantage in military power and economic resources. Over the course of a peaceful rivalry, state *i*'s advantage continues to grow until it becomes economically and militarily dominant. State *j* enjoys some prosperity early on (peaking around $t \approx 10$); however, the drain from the competition quickly reduces state *j*'s economic resources, leading to its inability to maintain military capabilities. Clearly, a peaceful rivalry leaves state *j* powerless and vulnerable to state *i*. Peaces can be painful (Powell, 2006).

If we compare the results of the peaceful rivalry with the outcome from a counterforce war (the second column of Figure 1), we see that state *j* is better off moving from a cold war to a military conflict (but not to an industrial war).[13] In this simulation, state

[11]See Kadera (1998 and 2001) for other examples of simulations of differential equations in international relations research.

[12]There is no agreed-upon standard for the minimum number of simulations required to test a model. Reviewing past published works using this method shows a wide range in the total number of simulations, from a low of 15 to a high of 50. Unless the initial conditions are some how biased (non-random) there is no grounds for assuming more simulations would provide for different outcomes. Similar arguments are used regarding sample size in regression analyses.

[13]We can also compare the industrial war results with those for counterforce war and for peacetime rivalry. In the last column of Figure 1, the simulations indicate that *j* is no better off than it was in the other two scenarios. Its initial resource advantage (which comes about from reverse-time simulations) erodes, so it is left the underdog both economically and militarily.

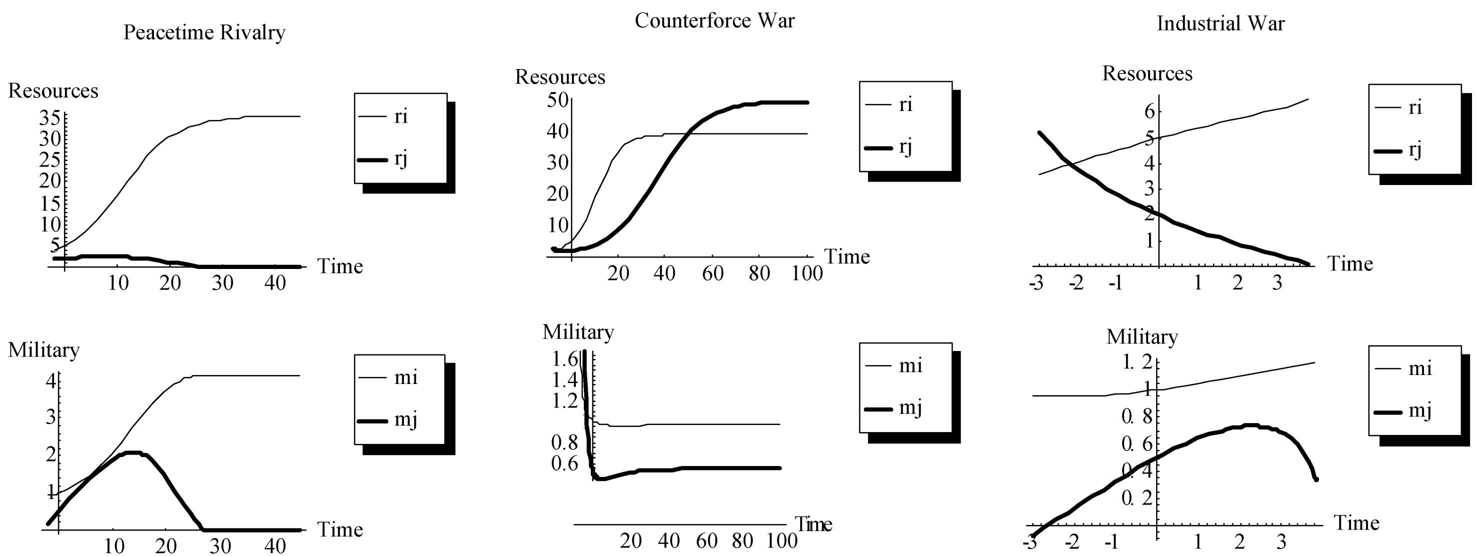

FIGURE 1 Simulations for initial conditions where $r_i(0) = 5$, $r_j(0) = 2$, $m_i(0) = 1$, $m_j(0) = 0.5$.

i still enjoys an initial advantage in economic resources and military capability, and this advantage grows over the early part of the conflict. However, state *j* is able to slowly catch and surpass state *j* in economic resources. In this case, state *j* survives through fighting (resources and military spending values do not go to zero), and it actually gains an advantage over state *i* in aggregate resources ($r_j > r_i$). This resource advantage is tempered some by the fact that state *j* remains militarily weaker than state *i*, but not by a large amount.

In all of the simulations conducted so far, this is the only scenario in which a state prefers traditional warfare to peaceful competition. If a state is weaker both economically and militarily, its only hope for survival, or for being able to compete with another power, is through engaging in military conflict. We should note, however, that military conflict does not always allow the weaker state to surpass the stronger. In many simulations the weaker state loses a military conflict just as it would lose the peaceful rivalry. Moreover, an initially weaker state may gain an advantage in overall resources by fighting a traditional war, but still be unfavored in the military balance (as is the case for nation *j* in Figure 1). Alternatively, the initially weaker state may establish a military edge by fighting a counterforce war but also lose the resources competition (as is the case for nation *i* in Figures 1 and 2). This means that being economically and militarily inferior is not sufficient to cause a war. The simulations only indicate that some states may be better served by fighting, as it is their only chance of winning a competition, of some type, with a superior foe (even if the odds are slim). While war may not be a certain victory, peace is a certain defeat (initially weaker states often face extinction over the course of a peaceful rivalry), which should be enough to make state *j* prefer to fight. In other words:

Deduction 1: A state will only prefer war when it has fewer resources and a weaker military than does its opponent.

This is a very counterintuitive finding as it postulates that only the smaller (disadvantaged) state in a rivalrous relationship should find war to its advantage. At first blush, it would seem more logical that the stronger state would prefer war because it would be in a better position to win. In some ways, Deduction 1 meshes with power transition theory's prediction that the weaker state initiates conflict (Organski & Kugler 1980).[14] The fighting-versus-investing model's finding refines the Power Transition prediction by specifying that the state must be both militarily and economically weaker than the other state, and there does not need to be a transition (or a possibility of one). In the two examples presented above, state *j* has a higher carrying capacity, meaning that state *i* was not in a position to overtake state *j* (a permanent transition was not possible). Simply being the smaller party in a competitive relationship can be enough to provide states an advantage from fighting.

Military Stalemate

A second prediction of the model is that under most conditions, traditional military wars will settle into long-term equilibria of suffering. In this disturbing scenario, neither state is able to eliminate the other (by driving its resources to zero); instead, each state's military spending and resources stabilize, and the opponents enter into a long-term war of attrition (see Smith, 1998) that neither side wins.

In the second column of Figure 2 (and the second columns of Figures 1 and 3), the two states clearly settle into a permanent level of conflict. While maintaining hostilities, each

[14]Organski and Kugler (1980) find that the rising power is more likely to initiate conflict to speed the transition.

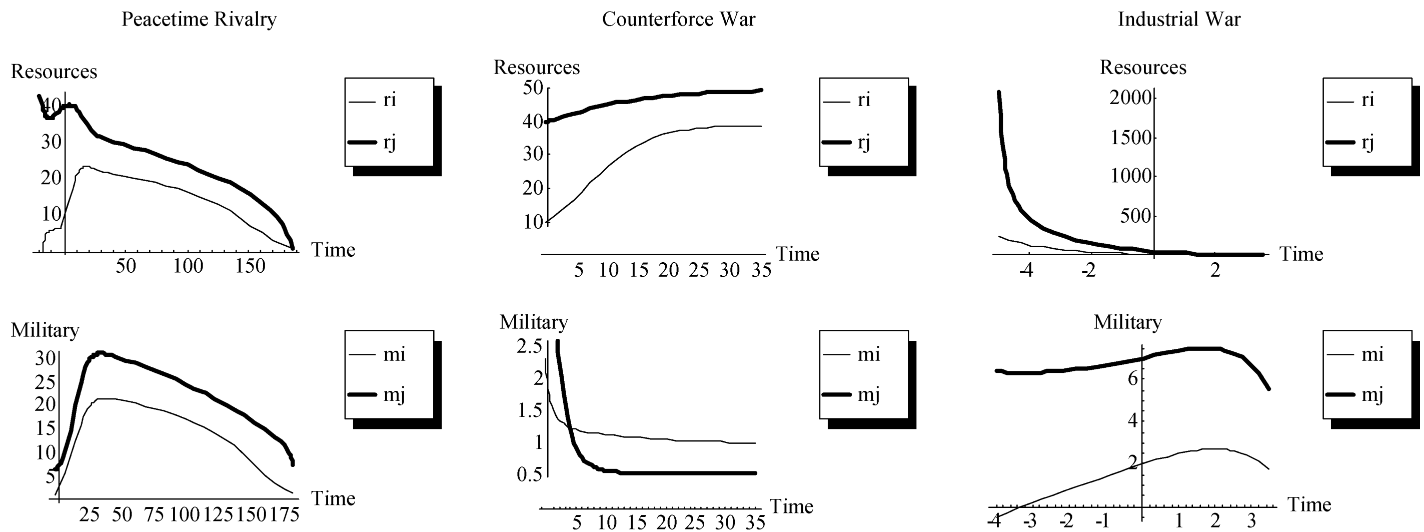

FIGURE 2 Simulations for initial conditions where $r_i(0) = 10$, $r_j(0) = 40$, $m_i(0) = 2$, $m_j(0) = 7$.

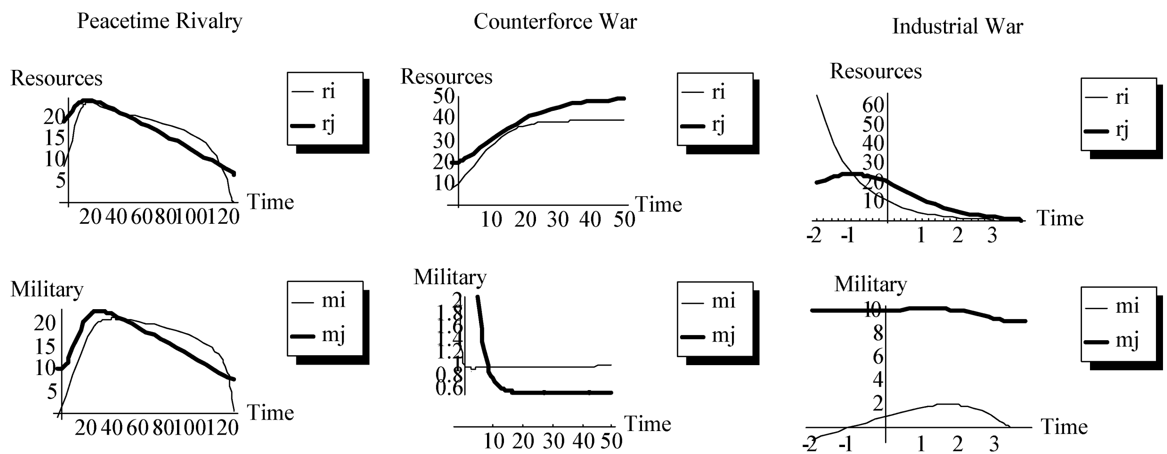

FIGURE 3 Simulations for initial conditions where $r_i(0) = 10$, $r_j(0) = 20$, $m_i(0) = 1$, $m_j(0) = 10$.

state is able to continue economic growth at a reasonably healthy rate before growth levels off and economic resources reach their maxima. In this case (as with the counterforce wars in Figures 1 and 3), the war evolves into a stalemate. Each side is still targeting military hostility towards the other and destroying the enemy's military resources, which is why m_i and m_j decline over the course of the simulation. Both states are nonetheless eventually able to replace the military power that is being destroyed from the fighting without significantly harming their economies. The traditional war trajectories demonstrate that each state is capable of continuing the conflict indefinitely and neither is able to gain an advantage over the other. When states are only able to engage in counterforce conflicts, the model predicts stalemate as the ultimate outcome:

Deduction 2: Counterforce wars result in stalemates.

The Iran-Iraq War appears to be an example of a pure counterforce conflict, one that also ended in a stalemate. Neither Iran nor Iraq could directly attack the other's economic resources, at least not in a sustained manner. Instead, they remained locked in a gruesome military stalemate that cost thousands of lives on both sides. The war did not end with either side truly defeated, as both were still able to maintain hostilities and neither had reached an economic breaking point. Only the realization of a deadlock and an aversion to further loss for no gain eventually ended this conflict.

Knowing that fighting a traditional military war can only lead to stalemate, under most circumstances, it seems logical that states would desire to avoid such an outcome. And if they are going to fight, they would prefer to target the industrial resources of the other state. However, states might not always be able to attack their enemy's resource bases (Slantchev, 2003). The Allies in the First World War did not have the force projection capability that the bomber provided in the Second World War. Instead, they had to settle for a slow blockade of Germany. In the Iran-Iraq War, neither antagonist possessed the ability to target the rear areas of the other on a large and consistent scale; they simply lacked the military capability to do so. Attrition can also be seen following MacArthur's departure from the Korean theatre (Gartner & Myers, 1995, 382). U.S. strategy became constrained by the 38th parallel, which limited its efforts to the imposition of enemy casualties instead of targeting the infrastructure used by China to resupply weapons and equipment. At the 1951 Congressional hearings, MacArthur complained that U.S. military and political leaders had not even established the productive power of China: "... none of us know the capacity of the enemy. He may build faster than we do. I couldn't tell you." (U.S. Senate, Foreign Relations and Armed Services Committees Hearings, 1951, 82). Thus, competitors may become locked into painfully long wars with no hope of either side winning because they lack the destructive capacity necessary to fully target the enemy's industrial resources. In an ironic twist, states with less ability to harm each other will become locked into long-term struggles that may very well raise the cumulative amount of devastation.

War Duration

The final prediction from the model deals with the length of wars. The simulations unequivocally demonstrate that traditional military wars tend toward stalemate because neither side can reduce the enemy's ability to resupply its armed forces. However, as a state targets its opponent's economic base, the length of wars will decrease, and stalemates will be avoided.

Looking at the last column in Figure 3, we can see that the industrial war between states i and j lasts from $t = -2$ to just past $t = 3$. This is a very short span compared to the length of the peacetime rivalry, which exceeds $t = 120$, and a counterforce war that settles into a hurting stalemate (the simulation ends at $t = 50$). The pattern is the same if we look at

Figures 1 and 2 above. In every simulation of the model, industrial wars are much shorter than cold wars or counterforce wars. Therefore, we offer:

Deduction 3: Industrial wars lead to the shortest conflict duration.

Industrial wars are shorter because they reduce a participant's ability to continue fighting by eliminating its capability to produce more resources for battle. While industrial wars are shorter, the simulations also demonstrate that they are more immediately destructive to both states. In most of the simulations, both states near complete exhaustion (i.e., m and r approach zero), and then one is eliminated from the competition (because $m = 0$ or $r = 0$). Thus, resource wars are shorter but tend to produce a higher level of damage for the victor.[15] As states gain better military capabilities for targeting the homeland of other nations, or for executing "strategic bombing" (Overy, 1995, 106) of industrial targets, wars will become shorter.

Conclusions and Future Directions

This paper presented a mathematical model of the relationship of two states in peace competition and in overt military conflict. We drew from theories of international relations, conflict processes, and ecology to formulate the equations.

The fighting-versus-investing model demonstrates how long range forecasts about the consequences of allocation trade-offs anchor states' preferences for various methods of interstate competition. These forecasts, represented by the simulations results, highlight the trajectories along which rivals' aggregate resource bases and military capabilities will unfold. Dynamic projections demonstrate that by considering the long-term results of a peacetime rivalry, weaker states might conclude that their only hope (slim as it may be) of winning or surviving a rivalry lies in fighting a counterforce war. Fighting a war that it is unlikely to win makes sense if the outcome is survival. In this respect, weaker rivals prefer a battle "to the pain" (Westley, in *The Princess Bride*) instead of "to the death."

Our resource allocation model's projections also yield insight into why and how stalemates evolve. Violent confrontations in which the two sides uniquely focus on destruction of one another's military forces produce deadlocks. Each side settles into an equilibrium in which it resupplies its military at the rate at which the enemy destroys it, without suffering further economic losses. When neither combatant is able to inflict additional harm on the other, a stalemate results.

Last, the dynamic model proposed here explains the connection between the type of war that is fought and the war's duration. When adversaries uniquely target industrial production, wars are shorter than when they attack only military objectives. As a consequence, we expect that as states become more capable of damaging their opponents' economic resources (perhaps through technological gains in airstrike capabilities, for instance), wars should terminate more quickly.

The fighting-versus-investing model serves as a foundation for understanding resource allocation trade-offs, but there are still avenues for improvement. Most important is the incorporation of simultaneous targeting of industrial and military resources in war-fighting efforts. The current construction of the models treats each of these ideal types separately, which is not an accurate reflection of reality. Modeling these two types of wars together so that states can split their war effort between attacking military and resource targets

[15]This does assume that both states can directly attack the resources of the other power. It is possible for only one state to be able to fight an industrial war while the other must attempt to force a military victory. It is hoped that future alterations to the model will be able to incorporate this form of asymmetric advantage.

should provide more insight into conflict behavior and the consequences of war policy on post war conditions and over time patterns during war. Incorporating such a feature would allow us to make predictions (akin to Deduction 3) about war durations as a function of how much states focus on military versus industrial targets. Similarly, our model does not provide for scenarios in which states simultaneously fight and pursue nonmilitary means of competition.[16] In practice, a state engages in a combination of the various strategies. We leave these nuanced representations of reality for future work.

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[16]Hirshleifer's (1988) model accounts for the *degree to which* states appropriate resources from the enemy or produce economic goods, and might therefore serve as a template for how to incorporate mixed forms of competition into the model.

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APPENDIX 1 Parameter Values

Parameter	State i	State j
α	0.2	0.1
β	0.07	0.12
ρ	0.4	0.1
δ	0.1	0.22
K	40	50